Charged C-metric with conformally coupled scalar field

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Abstract

We present a generalisation of the charged C-metric conformally coupled with a scalar field in the presence of a cosmological constant. The solution is asymptotically flat or a constant curvature spacetime. The spacetime metric has the geometry of a usual charged C-metric with cosmological constant, where the mass and charge are equal. When the cosmological constant is absent it is found that the scalar field only blows up at the angular pole of the event horizon. The presence of the cosmological constant can generically render the scalar field regular where the metric is regular, pushing the singularity beyond the event horizon. For certain cases of enhanced acceleration with a negative cosmological constant, the conical singularity disappears alltogether and the scalar field is everywhere regular. The black hole is then rather a black string with its event horizon extending all the way to asymptotic infinity and providing itself the necessary acceleration

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1 Introduction

The C-metric is a static and axially symmetric solution of the Einstein equations. It has the symmetries of a Weyl metric (see the analysis in [1] including a cosmological constant) and belongs to the special Class of Petrov type D metrics just as most common black hole solutions of the vacuum. The solution describes two uniformly accelerated black holes in opposite directions. To compensate the mutual gravitational attraction of the black holes, a conical singularity develops in one of its angular poles. This nodal singularity, often present in multi-black hole Weyl metrics, was interpreted [2] as due to the presence of a strut in between keeping the black holes away, or as two strings from infinity pulling in each one of the black holes in such a way as to obtain an (unstable) static equilibrium. The strut or the strings lie along the symmetry axis and they can be seen as the cause (or the effect) of the uniform acceleration of the black hole pair. This nodal singularity of a charged C-metric can be removed by an appropriate transformation introducing an external electromagnetic field [3]. In this new exact Ernst C-metric solution the acceleration of the pair of oppositely charged black holes is provided by the Lorentz force associated to the external magnetic field. The geometrical properties and physical interpretation of the C-metric were further developed and discussed [4].

The C-metric can be generalised by introducing into the solution a NUT parameter, a rotation and a cosmological constant term Λ [5], while in [6] a dilaton field non-minimally coupled was included into the solution. A careful and ingenious mathematical reincarnation of this solution led to the five-dimensional black ring metric found by Emparan and Reall [7] which opened up a whole new subject in higher dimensional black holes [8]. The cosmological constant version of these higher dimensional black holes has been however obstensively eluding discovery [1], [9] (see however the approximative methods of [10]). The flat spinning C-metric was further studied [11, 12, 13] and in particular in [13], the flat spinning C-metric has been transformed into the Weyl form and interpreted as two uniformly accelerated spinning black holes connected by a strut.

The presence of a cosmological constant does not change the generic features of the C-metric. The cosmological constant length scale, l plays a complimentary role to the acceleration parameter A shifting the horizon positions. In particular, in the case of adS space there is a relation between the acceleration parameter A and l. The case $A < 1/l$ was studied in [14] while the case $A = 1/l$ was investigated in [15]. The case $A > 1/l$ was extensively studied in [16] where an analysis of the causal structure of the solution showed that it describes a pair of accelerated black holes in an adS background.

The Euclidean version of the C-metric has been used for pair creation of black holes. In [6] a generalization of the flat C-metric with a dilaton field, amended with a flat Ernst solution, to ensure a regular instanton, was applied in the context of quantum pair creation of black holes, that once created via quantum tunneling accelerate apart. Also the C-metric solution for generic Λ has been used [17, 18, 19] to describe the final state of the quantum process of pair creation of black holes. The quantum process that might create the pair would be the gravitational analogue of the Schwinger pair production of charged particles in an external electromagnetic field.

The usual form of the C-metric has a structure function which is a cubic polynomial

in its variable, while in the charged case the structure function is quadric. A new neat form of the charged C-metric was proposed in [20] in which the structure function after a coordinate transformation was written in a factorisable form. This form of the C-metric leads to considerable simplifications, intuitive understanding, and allows to cast the metric in Weyl coordinates. An analogous new form of the charged rotating C-metric was also proposed [21] with a factorisable structure function. The difference with the non-rotating case is that this new form of the metric is not related to the old one by a coordinate transformation.

In this work we will present a generalization of the charged C-metric with conformally coupled scalar hair. The metric solution is similar to the $M = Q$ charged C-metric[1] but includes a scalar and EM field which have electric, magnetic and scalar charge linked by one relation to the mass and cosmological constant. In the limit of zero acceleration and zero cosmological constant we find the conformally coupled scalar black hole solution of [25]. This solution has a regular metric horizon covering a trapped black hole geometry with a non-trivial scalar field. The scalar field however, blows up at the black hole's horizon in accordance to "no hair" expectations. If one allows for a non-zero cosmological constant one can push the scalar singularity behind the event horizon, and obtain a regular solution of a charged black hole conformally coupled to a scalar field as found in [22] and [23]. For the solution we find here, we will see that acceleration plays a similar role to a cosmological constant. We will find that for a choice of parameters the scalar field is regular at the horizon and beyond if $\Lambda \neq 0$ and it is singular at the conical tip of the horizon in the flat case. Furthermore, for negative cosmological constant and enhanced acceleration we will see that the accelerated black hole becomes much like a black string with its event horizon reaching out all the way to asymptotic infinity. This case follows closely the description of the constant vacuum adS C-metric given in [15]. We will see that the scalar is then well behaved in the whole of the permitted coordinate region and furthermore, the metric has no longer a conical singularity. Following [15] we can interpret this as the string being smoothed out by the accelerating black string itself, extending all the way to asymptotic infinity. In this sense the conical singularity is replaced by the black hole horizon itself! This effect therefore is not due to the scalar field as one could have naively originally thought. We present the solution in the next section and then transform to Hong-Teo [20] coordinates in order to conclude in the last section.

2 Accelerating Charged Black Hole Coupled to a Scalar Field

Consider the action

$$I = \int d^4x\sqrt{-g}\left[\frac{R-2\Lambda}{16\pi} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}R\phi^2 - \alpha\phi^4 - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}\right] \,, \qquad (2.1)$$

where α is a dimensionless constant. The corresponding field equations are ($G = c = 1$)

$$G_{\mu\nu} + \Lambda g_{\mu\nu} \;=\; 8\pi\left(T^{(S)}_{\mu\nu} + T^{(EM)}_{\mu\nu}\right) \,, \qquad (2.2)$$

[1]One is tempted to call this case extremal but here the inner Cauchy and event horizon do not coincide as for the RN metric.

$$\Box\phi = \frac{1}{6}R\phi + 4\alpha\phi^3 , \tag{2.3}$$
$$\partial_\nu[\sqrt{-g}F^{\mu\nu}] = 0 , \tag{2.4}$$

where the energy-momentum tensors of the scalar and electromagnetic fields are respectively,

$$T^{(S)}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{1}{6}\left[g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu}\right]\phi^2 - \alpha g_{\mu\nu}\phi^4 , \tag{2.5}$$
$$T^{(EM)}_{\mu\nu} = \frac{1}{4\pi}g^{\alpha\beta}\left[F_{\mu\alpha}F_{\nu\beta} - \frac{1}{4}g_{\mu\nu}g^{\gamma\delta}F_{\gamma\alpha}F_{\delta\beta}\right] , \tag{2.6}$$

and $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$.

The scalar field is non-trivially and conformally coupled, therefore the total energy-momentum tensor $T_{\mu\nu}$ is traceless, and so the Ricci scalar curvature is constant, $R = 4\Lambda$. In fact, since $T^{(S)}_{\mu\nu}$ is traceless, it will play, for particular couplings, the same role as the EM energy-momentum tensor $T^{(EM)}_{\mu\nu}$. The field equations (2.2)-(2.4) admit the following solution,

$$ds^2 = \frac{1}{A^2(x-y)^2}\left[F(y)dt^2 - \frac{1}{F(y)}dy^2 + \frac{1}{G(x)}dx^2 + G(x)d\varphi^2\right] , \tag{2.7}$$
$$F(y) = \frac{\Lambda}{3A^2} + 1 - y^2 - 2mAy^3 - m^2A^2y^4 , \tag{2.8}$$
$$G(x) = 1 - x^2 - 2mAx^3 - m^2A^2x^4 , \tag{2.9}$$
$$\phi(y,x) = \sqrt{-\frac{\Lambda}{6\alpha}}\,\frac{Am(x-y)}{1+Am(x+y)} , \tag{2.10}$$
$$\mathcal{A} = eydt + gxd\varphi , \tag{2.11}$$

where e and g are the electric and magnetic parameters respectively and they are related to the mass through the relation

$$e^2 + g^2 = m^2\left(1 + \frac{2\pi\Lambda}{9\alpha}\right) . \tag{2.12}$$

The geometry of the black hole (2.7) is the same as the charged C-metric, with the only difference that the ratio of the electromagnetic charges to mass has a bound as can be seen in (2.12) and that the usual C-metric polynomials F and G are identical to the charged electric C metric of mass M and charge Q for $M = Q$. In fact, under (2.12), it would seem that the scalar and EM field of each black hole act merely as test fields in the $Q = M$ black hole geometry, allowing a static equilibrium to be obtained.

We can also write the solution in the Einstein frame. Permorming a conformal transformation, with a scalar redefinition of the form

$$\widetilde{g}_{\mu\nu} = \left(1 - \frac{4\pi}{3}\phi^2\right)g_{\mu\nu} \quad , \quad \Psi = \sqrt{\frac{3}{4\pi}}\mathrm{Arctanh}\left(\sqrt{\frac{4\pi}{3}}\phi\right) , \tag{2.13}$$

the action (2.1) becomes

$$I = \int d^4x\sqrt{-\widetilde{g}}\left[\frac{\widetilde{R}-2\Lambda}{16\pi} - \frac{1}{2}\widetilde{g}^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi - V(\Psi) - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}\right] , \tag{2.14}$$

where the self-interaction potential is

$$V(\Psi) = \frac{\Lambda}{8\pi}\left[\cosh^4\left(\sqrt{\frac{4\pi}{3}}\Psi\right) + \frac{9\alpha}{2\pi\Lambda}\sinh^4\left(\sqrt{\frac{4\pi}{3}}\Psi\right) - 1\right] . \tag{2.15}$$

In this frame the solution takes the form

$$ds^2 = \frac{u(y,x)}{A^2(x-y)^2}\left[F(y)dt^2 - \frac{1}{F(y)}dy^2 + \frac{1}{G(x)}dx^2 + G(x)dz^2\right] , \tag{2.16}$$

$$u(y,x) = 1 + \frac{2\pi\Lambda}{9\alpha}\left(\frac{Am(x-y)}{1+Am(x+y)}\right)^2 , \tag{2.17}$$

$$\Psi(y,x) = \sqrt{\frac{3}{4\pi}}\mathrm{Arctanh}\left(\sqrt{-\frac{2\pi\Lambda}{9\alpha}}\,\frac{Am(x-y)}{1+Am(x+y)}\right) , \tag{2.18}$$

with $F(y)$, $G(x)$ and $\mathcal{A}$ given by (2.8), (2.9) and (2.11). In this frame the scalar Ψ is minimally coupled with the expense of having a precise self-interaction scalar potential rather than a simple cosmological constant in the action. The limit of $\Lambda \to 0$ is obtained upon letting the coupling $\alpha \to 0$ so that $\frac{\alpha}{\Lambda} \sim$ *constant*. This solution then reduces smoothly to the solution found in [25] for zero acceleration $A = 0$. When $A \neq 0$ but $\Lambda = 0$ the potential drops out giving the solution of [6] at minimal EM-scalar coupling. If finally on the other hand $\Lambda \neq 0$, we obtain the solutions found in [22], [23], [24].

2.1 Properties of the Solution

The solution we have presented will follow closely the properties of the $Q = M$ charged C-metric, (see for example [16]). Here, we give the generic properties and we concentrate on the possible regularity of the scalar field. This is in part relevant to the no-hair theorems for general relativity. In order for the metric (2.7) to have the right signature $G(x)$ must be positive whereas $F(y)$ has to be negative. For $Am \neq 0$ equation $G(x) = 0$ has up to four real roots

$$\xi_1 = \frac{-1-\sqrt{1+4Am}}{2Am} , \tag{2.19}$$

$$\xi_2 = \frac{-1-\sqrt{1-4Am}}{2Am} , \tag{2.20}$$

$$\xi_3 = \frac{-1+\sqrt{1-4Am}}{2Am} , \tag{2.21}$$

$$\xi_4 = \frac{-1+\sqrt{1+4Am}}{2Am} . \tag{2.22}$$

When $\Lambda = 0$ we have $F(\xi) = G(\xi)$ and the situation is relatively simple. When $\Lambda \neq 0$ the angular sections $y = constant$ are similar but the $x = constant$ sections are shifted around. To find the horizons we must solve the equation $F(y) = 0$, which a priori gives four solutions,

$$y_1 = \frac{-1 - \sqrt{1 + 4Am\sqrt{1 + \Lambda/3A^2}}}{2Am}, \tag{2.23}$$

$$y_2 = \frac{-1 - \sqrt{1 - 4Am\sqrt{1 + \Lambda/3A^2}}}{2Am}, \tag{2.24}$$

$$y_3 = \frac{-1 + \sqrt{1 - 4Am\sqrt{1 + \Lambda/3A^2}}}{2Am}, \tag{2.25}$$

$$y_4 = \frac{-1 + \sqrt{1 + 4Am\sqrt{1 + \Lambda/3A^2}}}{2Am}. \tag{2.26}$$

For the y_i to be real it is necessary to have $1 + \Lambda/3A^2 \geq 0$ which is always satisfied for $\Lambda \geq 0$ and for adS space the relation is satisfied if $\mid A \mid \; > 1/l$. Now y_2, y_3 are real provided that $Am \leq \frac{1}{4\sqrt{1+\Lambda/3A^2}}$ and y_1, y_4 are real when $Am \geq -\frac{1}{4\sqrt{1+\Lambda/3A^2}}$. At $y \to -\infty$ we have a curvature singularity, and therefore the usual radial coordinate is roughly speaking $r \sim -\frac{1}{y}$. Null and timelike infinity arises at the zero for the conformal factor at $x = y$. To have $G(x) > 0$, we restrict the x coordinate to the range (x_s, x_n), while y must belong to the range $-\infty \leq y < x$ for all $x \in (x_s, x_n)$. Depending on the value of Am we have the following cases:

Case I. Am > 0

Ia. Am $< \frac{1}{4}$

In this case the equation $G(x) = 0$ has four real roots, where we have, $\xi_4 > 0 > \xi_3 > \xi_2 > \xi_1$ and

$$G(x) > 0 \Rightarrow \begin{cases} \xi_3 < x < \xi_4 \\ \quad \text{or} \\ \xi_1 < x < \xi_2 \end{cases}$$

We a priori concentrate on the interval $\xi_3 \leq x \leq \xi_4$ for the metric to have Lorentz signature. The axis $x = \xi_3$ points towards spatial infinity, and the axis $x = \xi_4$ points towards the other black hole. The surface $y = y_1$ is the inner black hole horizon, $y = y_2$ is the event horizon and $y = y_3$ the acceleration horizon. Now, as we noted earlier, for $\Lambda = 0$ the roots of F and G coincide, therefore the spacelike y region is (ξ_2, ξ_3) and $y < x$ for all $\xi_3 \leq x \leq \xi_4$. Therefore the surfaces of constant y are topological spheres [26]. Allowing a positive cosmological constant shrinks the (y_2, y_3) interval, i.e $\xi_2 < y_2 < y_3 < \xi_3$ and thus the topology of the angular sections remains spherical. This is to be expected since a positive cosmological constant "enhances" the acceleration. In order for the event

and acceleration horizon to exist we have a lowest bound for the black hole mass namely, $\frac{1}{16m^2} \geq A^2 + \frac{\Lambda}{3}$. On the contrary for $\Lambda < 0$ we have $y_2 < \xi_2 < \xi_3 < y_3$ and whenever $y > \xi_3$ we only have one axis (since $y < x$) [15]. This means that there is a topology change as $y > \xi_3$ and the spaces of constant y are then topologically R^2. The horizon $y = y_3$ is now the adS horizon.

Let us now examine the behavior of the scalar field in the allowed range of x and y. What we are interested in, is the scalar field regularity. So we will examine the scalar field denominator. We have

$$\Sigma = 1 + Am(x+y) \geq 1 + Am(\xi_3 + y_2) = \frac{1}{2}\left(\sqrt{1-4Am} - \sqrt{1 - 4Am\sqrt{1+\frac{\Lambda}{3A^2}}}\right)$$

The locus of $\Sigma = 0$ is the region where the scalar field explodes. We see that for $\Lambda = 0$ we have $\Sigma \geq 0$ [2], for $\Lambda > 0$, Σ is positive everywhere, while for $\Lambda < 0$, Σ can be zero even before we reach the event horizon.

Ib. Am $\geq \frac{1}{4}$

In this case $\xi_4 > 0 > \xi_1$ and $G(x) > 0$ is a priori in the range $\xi_1 < x < \xi_4$ (for $Am = \frac{1}{4}$ we have $\xi_1 < \xi_2 = \xi_3 < \xi_4$). This means that all $\Lambda \geq 0$ solutions are singular since $F < 0$ to the left of $y = y_1$ and therefore we are exposed to the naked singularity. The only interesting case is for $\Lambda < 0$ where $y = y_2$ and $y = y_3$ can still be real. Here, we will have $y \leq x < \xi_4$ and spaces of constant y are topologically R^2 planes. In this case, [15] the event horizon can hit asymptotic infinity so the solution resembles a black string. However, it is easy to show that $\Sigma \geq 1 + 2Amy_2$ which always has a solution for $4Am \geq 1$ and therefore the scalar will be singular for $y_2 < y < y_3$.

Case II. Am < 0

IIa. Am $> -\frac{1}{4}$

In this case the equation $G(x) = 0$ has four real roots, where it holds: $\xi_2 > \xi_1 > \xi_4 > 0 > \xi_3$ and

$$G(x) > 0 \Rightarrow \begin{cases} \xi_3 < x < \xi_4 \\ \quad\text{or} \\ \xi_1 < x < \xi_2 \end{cases}$$

We restrict $\xi_1 \leq x \leq \xi_2$ in order for the metric to have Lorentz signature and to keep $y < x$. Null or timelike infinity is reached when $y \to x$. The axis $x = \xi_1$ points towards spatial infinity, and the axis $x = \xi_2$ points towards the other black hole. The surface $y = y_3$ is the inner black hole horizon, $y = y_4$ is the event horizon and $y = y_1$ is now the acceleration horizon.

[2] $\Sigma = 0$ at the horizon and positive outside, and as we will see in the next section the singularity is only at the one pole of $G(x)$.

We have for the denominator of the scalar:

$$\Sigma = 1 + Am(x+y) \geq 1 + Am(\xi_1 + y_4) = \frac{1}{2}\left(\sqrt{1 + 4Am\sqrt{1 + \frac{\Lambda}{3A^2}}} - \sqrt{1+4Am}\right)$$

which is now everywhere positive for $\Lambda < 0$, and it can be zero for $\Lambda > 0$, while for $\Lambda = 0$ it is zero again on the horizon. Therefore the situation here is reversed for the sign of the cosmological constant with respect to the case of $0 < Am < \frac{1}{4}$.

IIb. $\mathbf{Am \leq -\frac{1}{4}}$

In this case $\xi_2 > 0 > \xi_3$ and $G(x) > 0$ is a priori in the range $\xi_3 < x < \xi_2$ (for $Am = -\frac{1}{4}$ we have $\xi_3 < \xi_4 = \xi_1 < \xi_2$). This means that all $\Lambda \geq 0$ solutions are singular since $F < 0$ to the left of $y = y_3$ and therefore we are exposed to the naked singularity. The only interesting case is for $\Lambda < 0$ where $y = y_4$ and $y = y_1$ can still be real. In other words the situation is similar to case Ib. Here, we will have $y \leq x < \xi_2$ and spaces of constant y are topologically R^2 planes. Once again, [15] the event horizon reaches all the way up to asymptotic infinity at $y = x$, hence the solution resembles rather a black string. There is one important difference from Ia. Here, it is easy to show that $\Sigma \geq 1 + 2Amy_4 = \sqrt{1 + 4Am\sqrt{1 + \frac{\Lambda}{3A^2}}}$ which is always positive (and thus the scalar is always regular) in the permitted range of the coordinates

$$-\frac{1}{4\sqrt{1 + \frac{\Lambda}{3A^2}}} < Am \leq -\frac{1}{4}\ . \tag{2.27}$$

Furthermore, removal of the nodal singularity in the (x, φ) sector implies [16] that

$$G'(x_s) = -G'(x_n)\ . \tag{2.28}$$

We can easily check that the above relation is satisfied if $(x_s, x_n) = (\xi_2, \xi_3)$ and enhanced acceleration namely, Am is either greater than $1/4$ or less than $-1/4$ (cases Ib, IIb). It is in particular true in the range of (2.27). Therefore, the scalar field is regular and there are no conical singularities if Am is in the range of (2.27). Here we should emphasize that the absence of the conical singularity is not a particular feature of the presence of the scalar, rather this is true for the $Q = M$ charged C-metric. So how can it be that the black hole can accelerate without a string providing the acceleration? We noted here that the topology of the black hole event horizon changes from spherical to planar. Furthermore we saw that the horizon reaches all the way out to asymptotic infinity. Therefore the string providing the acceleration has been replaced by the black hole-string itself. The horizon is actually the thickened out regular string itself! The case (2.27) is the only case where, the metric and scalar field have C^2 regularity in the permitted coordinate region.

3 Factorisable Form of the Solution

To have a better understanding of the parameters involved, we can write the metric (2.7) in a more familiar form. To do so, it is more convenient first to bring the function

$G(x)$ in a factorisable form. Following [20] we write

$$G(x) = (1-x^2)(1+Ak_1x)(1+Ak_2x)\ , \tag{3.1}$$

and we make the following coordinate transformations

$$\begin{aligned} x &\rightarrow Bc_0(x-c_1)\ , && (3.2)\\ y &\rightarrow Bc_0(y-c_1)\ , && (3.3)\\ t &\rightarrow \frac{c_0}{B}t\ , && (3.4)\\ \varphi &\rightarrow \frac{c_0}{B}\varphi\ , && (3.5)\end{aligned}$$

where c_0, c_1, B are real constants. In order to preserve the form of the line element we must have

$$\begin{aligned} A &\rightarrow \frac{A}{B}\ ,\\ G(\xi) &\rightarrow B^2G(\xi)\ . \end{aligned} \tag{3.6}$$

From (3.6) we get:

$$\begin{aligned} m &= \frac{M}{c_0^2\sqrt{1+2AM}}\ , && (3.7)\\ c_0 &= \frac{1+AM+2AMc_1}{\sqrt{1+2AM}}\ , && (3.8)\\ c_1 &= \frac{-1-AM+\sqrt{1+2AM+5A^2M^2}}{2AM}\ , && (3.9)\\ B &= \sqrt{\frac{1+2AM}{1+2AM+A^2M^2}}\ , && (3.10)\\ k_2 &= \frac{M}{1+2AM}\ , && (3.11)\end{aligned}$$

and we have set $k_1 = M$.

Then, the function $G(x)$ of (3.1) is written

$$G(x) = (1-x^2)(1+AMx)\left(1+\frac{AM}{1+2AM}\,x\right)\ , \tag{3.12}$$

with roots

$$\xi_1 = 1 \quad , \quad \xi_2 = -1 \quad , \quad \xi_3 = -\frac{1}{AM} \quad , \quad \xi_4 = -\frac{1+2AM}{AM}\ . \tag{3.13}$$

Suppose $\xi_1 > \xi_2 > \xi_3 > \xi_4$ [3]. Then $G(x) > 0$ in the range $-1 < x < 1$ (or in $\xi_4 < x < \xi_3$). We can not remove both conical singularities at $x = -1$ and $x = +1$, at the same time, since we can easily check that the condition $G'(\xi_1) = -G'(\xi_2)$ does not hold.

[3] That is $0 < \frac{AM}{1+2AM} < AM < 1$

To bring the metric in a familiar form we can now set

$$y = -\frac{1}{Ar} , \tag{3.14}$$

$$x = \cos\theta . \tag{3.15}$$

Then, the solution is written in the form

$$ds^2 = \frac{1}{(1+A\,r\cos\theta)^2}\left[-\frac{f(r)}{r^2}dt^2 + \frac{r^2}{f(r)}dr^2 + \frac{r^2}{p(\theta)}d\theta^2 + r^2\sin^2\theta p(\theta)d\varphi^2\right] \tag{3.16}$$

$$f(r) = -\frac{\Lambda}{3}r^4 + \left(1 - A^2r^2\right)(r-M)\left(r - \frac{M}{1+2AM}\right) , \tag{3.17}$$

$$p(\theta) = (1+AM\cos\theta)\left(1 + \frac{AM}{1+2AM}\cos\theta\right) , \tag{3.18}$$

$$\phi(r,\theta) = \sqrt{-\frac{\Lambda}{6\alpha}}\,\frac{k(Ar\cos\theta+1)}{r+k(Ar\cos\theta-1)} , \tag{3.19}$$

$$\mathcal{A} = -\frac{e}{r}dt + g\cos\theta d\varphi , \tag{3.20}$$

where

$$k = \frac{M}{1+AM} . \tag{3.21}$$

The electric and magnetic parameters are connected with the mass through the relation

$$e^2 + g^2 = \frac{M^2}{1+2AM}\left(1 + \frac{2\pi\Lambda}{9\alpha}\right) . \tag{3.22}$$

The constants A, M, e, g represent the acceleration, mass, electric and magnetic charge of the solution respectively, as we can see by taking the appropriate limits.

For $A = 0$ and $g = 0$, $e = 0$ ($i.e.$ $\alpha = -2\pi\Lambda/9$) we find the solution of a four-dimensional black hole in dS space coupled with a scalar field found in [22], so M is the mass of the non-accelerating black hole. For $A = 0$, $g = 0$ we find the solution of a charged black hole coupled with a scalar field [24], so e is the electric charge, while if we also have $\Lambda = 0$ we find the solution of [25]. Finally, if there is no scalar field we have the charged C-metric with $e = M$ ([2], [16], [20]).

It is interesting to consider the case $\Lambda = e = g = 0$. Then the scalar field reads

$$\phi(r,\theta) = \sqrt{\frac{3}{4\pi}}\,\frac{k(Ar\cos\theta+1)}{r+k(Ar\cos\theta-1)} . \tag{3.23}$$

Remember that in the BBMB black hole [25] the scalar field blows up at the horizon. In our case for $r = r_+ = M$ we get

$$\phi(r_+,\theta) = \sqrt{\frac{3}{4\pi}}\,\frac{M(AM\cos\theta+1)}{M(1+AM)+M(AM\cos\theta-1)} , \tag{3.24}$$

$$\Rightarrow \phi(r_+,\theta) = \sqrt{\frac{3}{4\pi}}\,\frac{AM\cos\theta+1}{AM(1+\cos\theta)} . \tag{3.25}$$

So the scalar is regular at the horizon except at the point $\theta = \pi$, *i.e.* at the pole $x = -1$. It would have been desirable to remove both conical singularities with an external electric field, employing an Ernst-like mechanism. In this case, the scalar field would be regular everywhere.

Following the same procedure as before, we can bring the metric (3.16) in hyperbolic form. Define

$$y = -\frac{1}{Ar} , \tag{3.26}$$

$$x = \cosh\theta . \tag{3.27}$$

the solution can be written in the form

$$ds^2 = \frac{1}{(1 + A\,r\cosh\theta)^2}\left[-\frac{f(r)}{r^2}dt^2 + \frac{r^2}{f(r)}dr^2 + \frac{r^2}{p(\theta)}d\theta^2 + r^2\sinh^2\theta p(\theta)d\varphi^2\right] \tag{3.28}$$

$$f(r) = -\frac{\Lambda}{3}r^4 - \left(1 - A^2r^2\right)(r + M)\left(r + \frac{M}{1 + 2AM}\right) , \tag{3.29}$$

$$p(\theta) = (1 - AM\cosh\theta)\left(1 - \frac{AM}{1 + 2AM}\cosh\theta\right) , \tag{3.30}$$

$$\phi(r, \theta) = \sqrt{-\frac{\Lambda}{6\alpha}}\,\frac{k(Ar\cosh\theta + 1)}{r - k(Ar\cosh\theta - 1)} , \tag{3.31}$$

$$\mathcal{A} = -\frac{e}{r}dt + g\cosh\theta d\varphi , \tag{3.32}$$

where

$$k = \frac{M}{1 + AM} , \tag{3.33}$$

and the electric and magnetic charge are connected with the mass through the relation

$$e^2 + g^2 = -\frac{M^2}{1 + 2AM}\left(1 + \frac{2\pi\Lambda}{9\alpha}\right) . \tag{3.34}$$

For $A = 0$ and $g = e = 0$ (*i.e.* $\alpha = -2\pi\Lambda/9$) we obtain the solution found in [23].

4 Conclusion

We presented a novel solution of a charged C-metric coupled to a scalar field in the presence of a cosmological constant. The metric geometry is identical to the "extremal" charged C-metric the difference being that the charge to mass ratio is bounded. The solution can be asymptopically dS, adS and flat depending on the value of the cosmological constant. There is a range of parameters, where the scalar field is regular on the horizon and beyond. In particular for $\Lambda = 0$ we see that the scalar field blows up only at the pole of the horizon since acceleration hides the remaining singular surface. In a sense the proper acceleration of the black hole plays a similar role as the cosmological constant but not quite

enough to allow scalar hair! For a choice of parameters, the conical singularities at both poles are absent similarily to the $Q = M$ limit of the charged C-metric [2], [19]. We saw that in this case the adS black hole resembles a black string with the event horizon reaching out to spatial infinity and being of planar topology. In one case, IIb, the scalar field is both regular and the metric is free of conical singularities. The black hole is being pulled away by its proper black string which provides a regularisation of the nodal singularity. This situation is similar to the one described in [15] for the adS C-metric.

We have seen that a conformally coupled scalar field plays a very similar role as an EM energy momentum tensor as long as $Q = M$. It would be interesting to promote this observation to a genuine solution generating method for Weyl or Papapetrou metrics with the inevitable aim to produce rotating black holes with scalar charge.

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Note added in proof: When this work was in its final stage, we got informed that another group is working on a similar problem. We had email exchanges, and they told us that they were studying a general case including rotations. In their submitted paper [27], they studied only the static limit of their metric (eqs (2.3)-(2.6)) which coincides with ours (eqs (2.7)-(2.11)).

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